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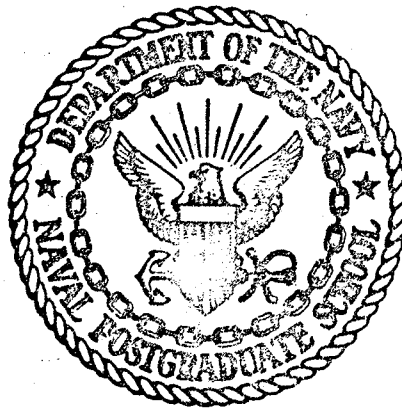
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Monterey, California



THESIS

PREDICTING AIRCRAFT EQUIPMENT REMOVALS
DURING INITIAL PROVISIONING PERIOD

by

Edwin August Fincke

September 1975

Thesis Advisor:

F. R. Richards

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The model is to be used before Fleet data are generated by obtaining aircraft-month estimates from the contractor and removal rate estimates from similar operational equipments. A probability distribution reflecting the degree of certainty is selected as a prior estimate. Then, as Fleet experience is accumulated the distribution is updated using Bayesian techniques and maturity growth curves. This distribution is used to give an estimate of current removal rate and to extrapolate to future removal rates.

Predicting Aircraft Equipment
Removals During Initial Provisioning Period

by

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Lieutenant Commander, United States Navy
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ABSTRACT

An investigation was made into the characteristics of program elements and removals of Weapon Replaceable Assemblies aggregated at the system level for the purpose of developing a method to predict removals during initial provisioning periods. From examination of nine avionic systems over a 28 month period a binomial model was developed using a removal rate based on aircraft-months as a program element.

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GLOSSARY OF TERMS

- System - A set of Weapon Replaceable Assemblies so related as to function as a unit. Identified by a four digit work unit code.
- WRA - (Weapon Replaceable Assembly). A major subassembly of a system. It can be removed and replaced by an Organization Maintenance Activity. For repairs it is sent to an Intermediate Maintenance Activity. Identified by a five digit work unit code.
- Naval Support Date - is the day that the Navy assumes responsibility for all support of a new system.
- Material Support Date - is the day that the Navy assumes responsibility for spares and repair parts support for a new system.
- Maintenance Action - All maintenance actions with action taken codes of A, C, J, K, and R.
- Removal - All maintenance actions with an action-taken code of R, a malfunction code other than 800, 801, 803, 804, 805, and 806 and a 10-digit job control number.
- Failure - All removals with a malfunction code other than 797, 798 or 799.
- False Removals - All removals with a malfunction code of 797, 798 or 799.
- BCM - Beyond Capability of Maintenance at the Intermediate Maintenance Level. All maintenance actions with action-taken codes of 1, 2, 3, and 6.

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I. INTRODUCTION

A. PROBLEM DESCRIPTION

When a new aircraft is being purchased, decisions on the quantities of spare parts to be bought are made even though information on expected demands, operating programs and the final configuration of the aircraft is limited. The initial provisioning period is normally eighteen months beginning from the Naval Support Date or Material Support Date. After this period, provisioning is based on usage data. Many projects have been conducted to improve the Naval Air Systems Command's ability to initially provision Weapon Replaceable Assemblies (WRA's) more effectively. One set of data that is common and critical to all these projects consists of removal or failure rates that can be expected when the equipment is initially introduced into Fleet operation. Since there exists no operational experience with the aircraft, prediction of initial provisioning levels have for the most part, differed by orders of magnitude from what experience suggested they should have been. This difficulty results directly from early provisioning procedures and cannot be avoided if spare WRA's are to be delivered concurrently with the aircraft from the very start.

There are ways, however, of alleviating this difficulty. It is possible to gather performance data from the experience gained in testing components and prototypes of the new aircraft. Characteristics of materials of manufacture, operations and

maintenance schedules are determined entirely independently from any demand experience. These considerations seemed to indicate the possibility of predicting demands from a priori information.

It should also be possible to gain data from the early experience of the first squadron in operation. Such data would in all probability be insufficient to predict future performances with any degree of confidence. Yet, they might permit a tentative sorting out of parts into broad demand categories - fast moving and slow moving. The collection of this early-experience data might help in guiding further provisioning.

This thesis is primarily concerned with removal rates because increased accuracy in predicting the number of removals during the provisioning period is needed to establish more realistic initial inventory levels. Removal, failure and Beyond Capability of Maintenance (BCM) actions are all of concern when provisioning WRA's. However, the action which triggers an issue from the supply storeroom is the removal. When a WRA cannot be repaired on the aircraft, it is removed and replaced with a ready-for-issue WRA. Thus, data on all three actions were collected, but this thesis analyses only the removal actions. The impact of removals as a key factor is illustrated in Figure 1.

SIMPLIFIED WRA FLOW

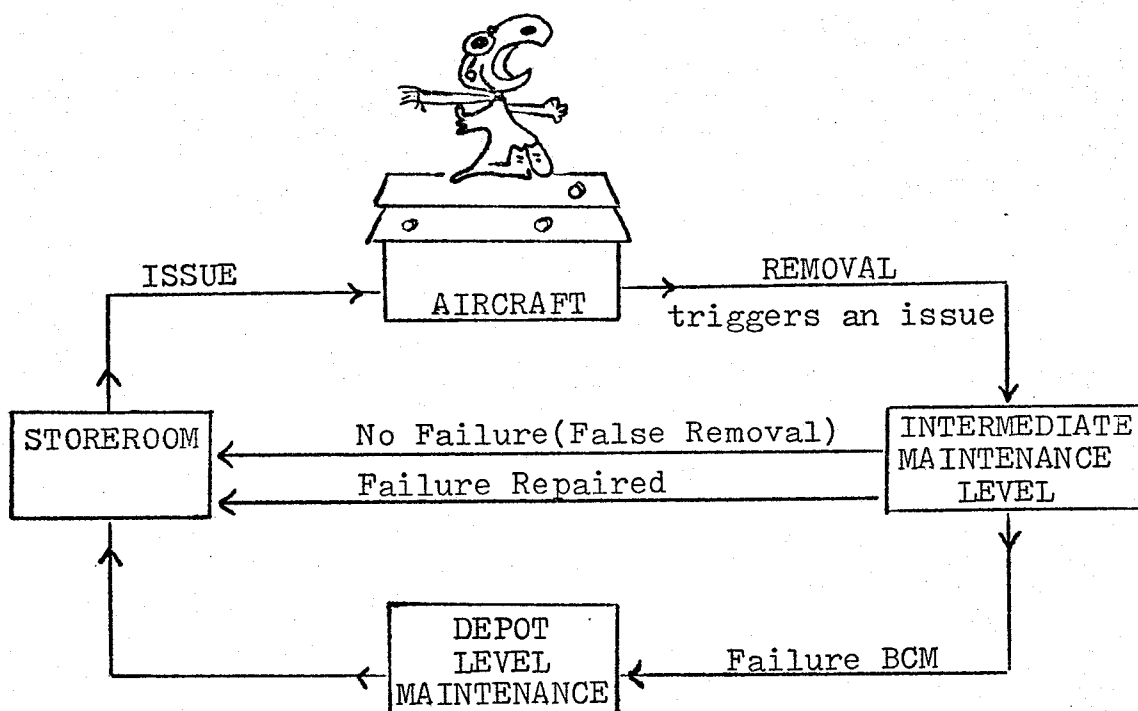


Figure 1

B. CURRENT METHODS OF PREDICTING INITIAL FAILURE/REMOVAL RATES

Current methods of predicting removal/failure rates are delineated in Ref. 1 and call for the contractor to obtain the Mean Time Between Failures (MTBF) for the WRA. This is accomplished by performing failure analysis to compute a stress number for each component of the Shop Repairable Assemblies (SRA's) contained in the WRA. These stress numbers are used as points of entry into MIL-HDBK-217 (Reliability Stress and Failure Rate Data for Electronic Equipment) from whence a set of generic failure rates is extracted and then multiplied by an appropriate environmental factor. These

products are then summed over components of the SRA's contained in the WRA and the result divided into 1,000,000 to obtain the estimated MTBF for the WRA.

The contractor then presents these MTBF's to the Naval Provisioning Team. The team then derates the MTBF's based usually on mature historical usage data of similar equipment. This adjusted MTBF is used to provision the equipment. To arrive at a final provisioning number, the adjusted MTBF is transformed to comply with current provisioning practices of incorporating flight hours, maintenance cycle base and rotatable pool factors.

C. OBJECT OF THE THESIS

The object of this thesis is to examine the nature of removals and program elements in order to develop a method to predict the number of removals of WRA's for initial provisioning purposes. This thesis is based upon the premise that, to the extent to which experience with earlier WRA's may be valid for a new WRA, it should be possible to estimate the general magnitude of removal demands, and then with the collection of early-experience data to refine these estimates.

II. DATA COLLECTION

A. SYSTEM SELECTION AND DATA EXTRACTION

1. System Level of Indenture

The system level of indenture was selected for data extraction even though naval provisioning in general does not provide spare systems. This selection was based on research accomplished at the Naval Aviation Integrated Logistic Support Center (NAILSC) which showed that if maturity growth curves were obtained and a prediction method was derived at the system level, then the same general procedure could be conducted at the WRA level [1]. This approach permitted a broad evaluation over several systems aggregating many WRA's. The same resources applied at the WRA level of indenture would have precluded such an examination.

2. Selection of Systems

Criteria used in selecting systems to be studied were the same as used in the NAILSC research [2].

a. Systems must be peculiar to specific aircraft to avoid confounding the data with respect to systems and airframes.

b. Systems must be mission essential so that if the system is not operational then the aircraft does not fly. Thus, the flight operating time on the system always equals the flight time.

c. The data base must include monthly performance data (e.g. removals, failures, BCM's) and utilization data (e.g. flight hours, sorties, number of reporting aircraft) for at least eighteen months starting with the Naval Support Data or Material Support Data or date of introduction into the Fleet.

Two non-avionic systems were chosen since, in about half the cases, operational readiness is degraded by these systems.¹ Although WRA removal rate prediction usually has been in the avionics area, it was instructive to compare the characteristics of non-avionics and avionics data to determine if predictive techniques for avionics could be applied to non-avionics.

The two non-avionics systems selected were the wing sweep hydraulic system in the F-14A aircraft and the hydraulic power system in the EA-6B aircraft which are further described in Appendix A. Five avionic systems in the A-7E aircraft and four avionic systems in the P-3C aircraft were selected and are described in Appendix C.

3. Data Extraction

Data were obtained from the Naval Maintenance, Material and Management System (3M). Utilization data account for the number of identical systems installed in the aircraft.

¹For example, the Aircraft Degradation Ranking Summary dated 23 August 1974 summarized Not Operationally Ready and Reduced Material Condition hours for the period January 1974 to June 1974 and reported that of the assemblies listed, 58% were non-avionic while 42% were avionic.

For example, effective number of sorties equals the number of sorties multiplied by the number of times a particular system is installed. No system selected in this thesis is duplicated on the aircraft so that the multiplier is one. On the WRA level of indenture, however, identical assemblies are frequently installed.

a. Non-Avionic Systems

The 3M System data in final form revealed many inaccuracies in the early stages of fleet introduction. Thus, Maintenance Action Form data by Work Unit Code were extracted from the 3M data base. From this computer printout inconsistencies were visible by following the Job Control Number so that data could be purified, then tabulated as presented in Appendix B. Utilization data were extracted from monthly 3M summaries and are displayed in Appendix B.

b. Avionic Systems

Data for the nine avionic systems were extracted from the 3M system by the NAILSC [2] and are tabulated in Appendix D.

B. COMPARISON OF AVIONIC AND NON-AVIONIC DATA

The number of removals, failures and BCM's experienced by the two hydraulic systems is much lower than experienced by the avionic systems. Appendix A shows the paucity of removal activity at the system level. This low removal activity is because failures of hydraulic WRA's can usually be repaired in the aircraft so that the

WRA's do not have to be removed for repair. Avionic WRA's, on the other hand, are usually removed to be repaired at the Intermediate Maintenance Activity. They are built with connectors which facilitate removal and replacement, and testing after replacement is localized. Hydraulic WRA's are not made for easy removal and replacement is more difficult because the entire fluid system must be bled and tested.

The quantity (removals/flight hours) for each month was plotted as a function of time. These two curves were similar to the avionic maturity growth curves derived by the NAILSC [2]. However, the NAILSC conclusion that statistical characteristics of avionic systems can be generally applied to the associated WRA's may not be shown as readily in the case of hydraulic systems. There were not enough data points in the hydraulic systems to justify further analysis in support of improving initial provisioning prediction techniques. Accordingly, research continued on only the nine avionic systems.

III. DATA EXAMINATION

Chapter III describes the nature of the avionic systems' data and mentions factors which should be considered when attempting to predict removals. The model development in Chapter IV is based upon the characteristics of removals and program elements as described herein.

A. CHARACTERISTICS OF REMOVALS

1. Relationship With Failures and BCM's

The correlation coefficients in Table I indicate a strong relationship between removals (R) and failures (F) and a weaker relationship between removals (R) and BCM.

CORRELATION BETWEEN PERFORMANCE DATA

System	Correlation Between		
	Rand F	Rand BCM	F and BCM
A7E			
1	.99	.54	.62
2	.96	.81	.85
3	.99	.86	.88
4	.98	.57	.59
5	.90	.61	.81
P3A			
6	.95	.77	.76
7	.88	.67	.80
8	.79	.54	.71
9	.94	.58	.68

Table I

Plotting the ratio (failures/removals) as a function of time generates a maturity growth curve that approaches a constant value. It appears, therefore, that given perfect information on the number of failures, the number of removals could be predicted.

2. Removals Over Time

The number of removals pass from a generally low level with the initial introduction of a few aircraft into the Fleet to a higher level. The early months frequently have zero demands which have relatively little informational value for statistical purposes. Fluctuations in the number demanded as time increases are apt to be large in absolute magnitude, but fluctuations of a given magnitude represent smaller percentage of variation.

It is clear that the aggregation of WRA removal demands necessarily implies that average system demand will be higher. The fluctuation in demands at the system level from month to month is a smaller relative value than at the WRA level. Hence, demand can be expected to track closer to its average value in system data than in single WRA data.

3. Fluctuations in Removals

Fluctuations in removals make prediction more difficult. This is especially so because the fluctuations are the result of three quite disparate factors. Some fluctuations are related with the variability in program elements (e.g. flying hours). This area will be discussed in Section B. Other fluctuations appear to be cyclical in nature, at least in systems with relatively high demand rates. Such cyclical fluctuations suggest that when a WRA is replaced on a large number of aircraft within a short period of time (e.g. a scheduled maintenance action), the demand for the WRA will

remain low for some time because of the small number of aircraft in which a new WRA has not been installed.

Finally, other fluctuations in demand are purely random in nature. These should be expected to even out over a sufficiently long period of time after Fleet introduction, but for WRA's with relatively low removal rates the period involved might be quite long.

Over long periods of time there may be changes in the level of demand for a family or system of WRA's resulting from persisting influences. Such influences may include design changes which prolong the mean time between removals, improved maintenance practices and operating procedures as Fleet personnel become more familiar with the new aircraft, or changes in the mission or deployment of the aircraft. Because of their persisting nature, these influences should be taken into account in making such predictions.

4. Non-Recurrent Events and Extrapolation

While fluctuations in removals over time tend to obscure the relations between removals and program elements, they were still subjected to statistical analysis as described in Section B. Events which are by their nature non-recurrent, although they may occur at specified times and in a planned manner, are inherently unrelated to any program element. Hence their occurrence cannot be predicted solely from a knowledge of the operational program for the aircraft. An example of a non-recurrent event is a modification program

which causes an unrepresentative demand or a design change which changes the average time between removals.

If removal rate data are used indiscriminately to predict removals, or if predictions are made before the occurrence of a non-recurrent event and no allowance is made for its influence, serious inaccuracies are almost certain to result. Prediction through extrapolation of data based on a functional relationship between removals and some program element relies on the assumption that the conditions under which the data are generated and the underlying relationships between dependent and independent variables are unmodified as between past and future periods of observation. The presence of non-recurrent events in either period invalidates this fundamental assumption.

The significance of non-recurrent influences is that they obscure such basic relationships as may exist between removals and program elements.

B. CHARACTERISTICS OF PROGRAM ELEMENTS

1. Relationship of Program Elements and Removals

There are many program elements such as flying hours, sorties, engine hours, fuel consumption, aircraft-months, time between flights which could influence the number of removals. References 3, 4, 5, and 6 indicate that demand for aircraft spare parts typically has no simple linear relationship with program elements. Reference 7 showed that historical groupings of flight hours and sorties could not

be utilized to yield accurate jet engine failure forecasts. Nothing was found in the literature to document that these observations would apply to removals of WRA's as well. The data analysis reported in this section suggests a simple linear relationship exists between removals of WRA's and program elements.

To test for this linear relationship, four simple linear regressions were performed on each of two systems, using the SNAP/IEDA computer statistical package. In each regression the dependent variable was removals (R) and the independent variable was one of the following program elements, flight hours (FH), sorties (S), number of reporting aircraft (AC) or aircraft-months (ACM).

Aircraft-months for month t (ACM_t) were calculated by multiplying the number of new aircraft reporting in month i ($i \leq t$), X_i , by the age of these aircraft in months which is $(t+1-i)$ and summing over all $i \leq t$. That is, $ACM_t = \sum_{i=1}^t (t+1-i)X_i$. For example, if the number of new A7E aircraft reporting for the first three months is 7, 7, and 16, then, $ACM_1 = (1)(7) = 7$; $ACM_2 = (2)(7) + (1)(7) = 21$; $ACM_3 = (3)(7) + (2)(7) + (1)(16) = 51$.

A Coefficient of Determination, which is a measure of the goodness of fit of the models to the data that shows the proportion of the variance of removal data explained by the model, was computed for each model. These Coefficients of Determination are displayed in Table II along with standard errors of the coefficients of the variables (shown in parentheses directly below the coefficients).

SIMPLE LINEAR REGRESSION MODELS
FOR SYSTEMS ONE AND SIX

<u>System #</u>	<u>Regression Equation</u>	<u>Coefficient of Determination</u>
1	$R = -29.5 + .049 \text{ FH}$ (.004)	.868
1	$R = -28.3 + .088 \text{ S}$ (.007)	.848
1	$R = -27.6 + 1.59 \text{ AC}$ (.139)	.834
1	$R = 80.8 + .113 \text{ ACM}$ (.008)	.882
6	$R = 10.3 + .005 \text{ FH}$ (.001)	.688
6	$R = 6.82 + .027 \text{ S}$ (.003)	.733
6	$R = 3.46 + .651 \text{ AC}$ (.071)	.763
6	$R = 13.4 + .042 \text{ ACM}$ (.006)	.651

Table II

Next, stepwise multiple linear regression was performed on all nine systems as another method of testing the strength of the relationship between removals and program elements. Here, sorties (S), number of reporting aircraft (AC) and the number of aircraft months (ACM) were all considered to be included in the model. The stepwise multiple linear regression mathematically chooses the variables that are most instrumental in explaining the variation of the data of the model. A coefficient is computed for the single most important variable and then the next most important variable is chosen. This

process continues until all variables have been considered or until the contribution of the next variable is not significant in explaining the remaining variation. Table III presents the coefficients for each of the program elements and the Coefficient of Determination for each of the nine regression equations. The standard deviations of the coefficients of the variables are shown in the parentheses.

MULTIPLE LINEAR REGRESSION MODELS
FOR NINE SYSTEMS

System #	Regression Equation	Coefficient of Determination
1	$R = 17.3 + .040S + 0 AC + .068ACM$ (.011) (.014)	.922
2	$R = 1.92 + .020S + 0 AC + 0 ACM$ (.002)	.846
3	$R = .670 + 0 S + .367AC + .025ACM$ (.091) (.006)	.915
4	$R = 14.0 + 0 S + .308AC + 0 ACM$ (.028)	.825
5	$R = - 6.95 + 0 S + .521AC + 0 ACM$ (.035)	.893
6	$R = 3.46 + 0 S + .651AC + 0 ACM$ (.071)	.763
7	$R = 22.7 + .034S + 0 AC + 0 ACM$ (.005)	.645
8	$R = 58.2 + .028S + 0 AC + 0 ACM$ (.010)	.248
9	$R = 9.13 + .059S + 0 AC + 0 ACM$ (.005)	.821

Table III

The residuals for each regression shown in Tables II and III were plotted over predicted values. A visual assessment of the plots was made to check on the regression assumptions of linearity and homoscedasticity [8]. The plots were approximately uniform in scatter and there was no evidence of curvilinearity except in the simple regressions of system #6 using AC and ACM and in the multiple regression of system #4. The Coefficients of Determination in Tables II and III indicate that most of the variance of the observed data is explained by the models. Table IV shows a strong positive correlation between the various program elements.

CORRELATION BETWEEN PROGRAM ELEMENTS
BY AIRCRAFT

A7E					P3C				
	FH	S	AC	ACM		FH	S	AC	ACM
FH	1.00	.99	.94	.88	FH	1.00	.99	.95	.95
S		1.00	.95	.88	S		1.00	.95	.91
AC			1.00	.88	AC			1.00	.96
ACM				1.00	ACM				1.00

TABLE IV

This high correlation between the program elements accounts for the fact that the multiple regression equations often involved only a single program element. However, this collinearity does not alter the predictive power of the total regression equation [8].

Reference 2 studies the merits of using flight hours (FH) or sorties (S) as a parameter base by comparing the goodness of fit of curves using (removals/FH) versus (removals/S). For the systems investigated in this thesis, Table IV shows that the correlations between flight hours and sorties were 0.99 indicating a nearly linear relationship between the two program elements. In fact, for the A7E, the slope of the line was approximately 1.82 flying hours/sortie with small residual variance, and the slope was 4.42 flying hours/sortie with a somewhat larger residual variance for the P3C. Thus, for these systems, the question as to which is the better program element, FH or S, is moot.

The correlations between removals (R) and the various program elements are shown in Table V.

CORRELATION BETWEEN REMOVALS AND PROGRAM ELEMENTS
BY SYSTEM

System #		FH	S	AC	ACM
1	R	.93	.92	.91	.94
2	R	.91	.92	.91	.82
3	R	.91	.92	.93	.93
4	R	.88	.90	.91	.84
5	R	.89	.90	.95	.82
6	R	.83	.86	.87	.81
7	R	.74	.80	.79	.65
8	R	.42	.50	.48	.33
9	R	.91	.91	.87	.85

Table V

The regressions in Table II and the correlation coefficients in Table V indicate a significant linear relationship between removals and program elements. It appears therefore that a stronger linear relationship exists between removals of WRA's and program elements, for the systems examined herein than the literature suggests exists between failures of spare parts and program elements.

2. Choosing a Program Element as a Parameter Base

The strong relationship between removals and program elements justified seeking a program element upon which to base the removal rate (removals/program element). The high correlation between program elements reduced the need of identifying the effect of each element so long as the same correlations can be expected in the future. Thus, it seemed unnecessary to distinguish between the four program elements as far as their relationship to removals is concerned.

The high correlations among the four program elements (Table IV) and between removals and each of the program elements (Table V) suggest the likelihood that any one of the program elements would summarize much of the information contained in the others. For example, a program element such as the number of landings may in fact stand for several different activities, such as, hours out of hanger, pre-flight inspections, taxiing, take-offs, and so on, each of which will contribute a certain amount to the aggregate wear and tear on a WRA, but all of which are closely correlated in turn with the number of landings. Reference 3 indicates that aircraft-months, in

particular, serve to represent all the factors which influence removals of WRA's in a uniform manner over time, for each aircraft individually.

The partial correlations in Table VI exemplify the effect of each program element separately upon removals. The values are the partial correlations between removals (R) and program element j, holding program element k fixed. Partial correlation removes the effect of other program elements and results in correlation coefficients smaller than the simple correlation coefficients in Table V. The partial correlations of removals with various program elements holding ACM constant are generally low (.63, .57, .54) indicating that still more information could be gained using other program elements. The higher partial correlations of removals with ACM holding other program elements constant (.70, .70, .71) indicate that ACM gives additional information not given by other program elements.

PARTIAL CORRELATION FOR SYSTEM 1 BETWEEN
REMOVALS AND PROGRAM ELEMENT j
HOLDING PROGRAM ELEMENT k FIXED

		k			
		FH	S	AC	ACM
j	FH	-	.35	.53	.63
	S	-.01	-	.43	.57
	AC	.28	.29	-	.54
	ACM	.70	.70	.71	-

Table VI

Although there is a high correlation between ACM's and flying hours and sorties, this does not imply dependence between ACM's and aircraft utilization. Thus, an increase of flying activity, caused by an outbreak of hostilities for example, will not necessarily cause a commensurate increase in ACM's.

Program elements are statistically measurable quantities, exerting a continuing, recurrent influence on the removal of WRA's. They are thus associated with the programmed employment of the aircraft in some systematic way, even though they may not be scheduled in advance. The usefulness of employing a program element to predict future removals might therefore be quite limited unless the program element itself can be predicted accurately. Reference 5 reported that nine month projections of flying hours resulted in 60 per cent of the projections having errors of at least 40 per cent. Such poor projections of flight hours would cause estimates of removals to err to the same degree, even if the exact relationship between flight hours and removals were known. It is best, therefore, to use explanatory variables (program elements) which can themselves be predicted with greater certainty.

Since the number of aircraft to be delivered to the Navy is scheduled in the manufacturer's contract, aircraft-months can be forecasted accurately from the production schedule delivered in the contract. Thus, it would seem that, among the four examined program elements, aircraft-months would have the greatest amount of certainty.

3. Characteristics of Removals per Aircraft-Month

As discussed above, ACM seemed to be a viable program element on which to base an estimate of removals. The quantity, removals/ACM, was calculated each month for the nine systems and plotted as a function of time. For comparison, the quantities removals/sorties and removals/aircraft were similarly calculated and plotted.

The plots of removals/sorties and removals/aircraft generally showed a downward, non-linear trend. A more definable trend, however, was generally displayed in the plots of removals/ACM. Figure 2 characterizes the trend which appeared to be similar to the classic reliability growth curve. This type of curve accounts for changes in reliability due to design modifications and other corrective actions taken during the development and early deployment of new aircraft.

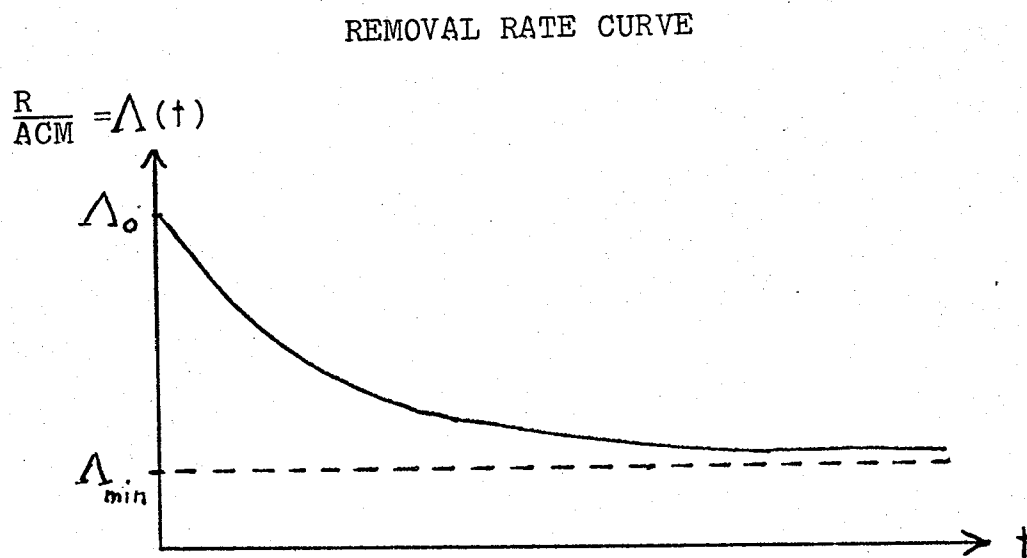


Figure 2

Figure 2 has been modeled [Ref. 9] as,

$$(R/ACM) = (\Lambda_o - \Lambda_{\min})e^{-bt} + \Lambda_{\min}$$

where Λ_o = initial removal rate
 Λ_{\min} = minimum removal rate

Since most systems experience a growth in reliability in the early stages of their life cycles it is natural to try to model removal rates using a decreasing failure rate (DFR) function. In the case of removal rate, $\Lambda(t)$ approaches Λ_{\min} asymptotically from above as t gets large. The parameter b can be interpreted to reflect the amount of management attention given to correcting defects in the WRA's. The larger the value of b , the more the reliability growth and the sooner the removal rate will reach its minimum level.

Plots of (R/ACM) over time typically reached the minimum level (Λ_{\min}) between the fifteenth and twenty-fifth month. Since initial provisioning is concerned with the first 18 months, the constant Λ_{\min} was not included in the model, thus simplifying the equation and facilitating regression analysis. The equation used to model the removal rate over time was,

$(R/ACM) = ae^{-bt}$, where $a = \Lambda_o$. The value of this model approaches zero as t gets large, so that extrapolation far into the future would not be appropriate. For the purpose of predicting the value of p during an 18 month initial provisioning period the model is satisfactory.

A logarithmic transformation was made yielding $\ln(R/ACM) = \ln a - bt$. Simple linear regression was used to estimate the parameters and to test if the coefficients of the curves were similar for all systems. Table VII shows the regression equations and indicates that the values for the parameter b have a narrow range. The Student's t test at a significance level of .05 failed to reject, in two-thirds of the tests, the hypothesis $H_0: b_i = b$, where b was taken as the overall mean value of the estimated slopes. (See Appendix E) The mean value for the five A7E systems was -0.062, with a standard deviations of -0.031, and for the four P3C systems the mean value was -0.115 with a standard deviation of -0.022.

The null hypothesis that $b=0$ was tested using the Student's t test and was rejected for eight of the nine equations at the .05 level of significance (system 4 was the exception) as shown in Appendix F. Thus, the b_i parameter estimates were seen to be significantly different from 0. The Coefficients of Determination were high, except for systems 4, 5, and 6. An examination of the residual plots against predicted values, however, revealed weak curvilinear trends. Thus, it could not be concluded that the regression assumptions of linearity and homoscedasticity were valid or that the residuals were statistically independent of each other. However, it has been shown in Ref. 10 that these curvilinear trends do not necessarily weaken the predictive power of the model if the ratio of degrees of freedom to number of

observations is very high. The removal rate model yields a ratio of $(26/28) = 0.93$. Thus, it seemed that the limited use of this model to make point estimates of future removal rates was valid.

SIMPLE LINEAR REGRESSION ON
 $\ln(R/ACM) = \ln a + bt$

System	Regression Equation $\ln(R/ACM) = \ln a + bt$	R^2
1	$\ln(R/ACM) = - .364 - .075 t$ (.007)	.802
2	$= - 1.241 - .103 t$ (.008)	.851
3	$= - 1.400 - 0.67 t$ (.007)	.792
4	$= - 3.325 - .022 t$ (.012)	.127
5	$= - 1.902 - .043 t$ (.012)	.349
6	$= - .861 - .090 t$ (.019)	.482
7	$= .296 - .118$ (.010)	.850
8	$= 1.215 - .143$ (.012)	.844
9	$= - .293 - .107$ (.010)	.803

Table VII

IV. MODEL DEVELOPMENT TO PREDICT REMOVALS

Chapter III discussed basic characteristics of removals and program elements and some fundamental considerations from examination of the data. In Chapter IV these principles and empirical relationships are applied to develop a model to be used to predict the number of removals per month.

A. THE MODEL

1. Binomial Model with a Bayesian Update

The task is to calculate the expected number of removals in month t , denoted $E [k_t]$. The model for predicting removals in a month assumes that the factors causing removals can be summarized by knowledge of the number of aircraft and the age of the aircraft operating in that month. This knowledge is represented by the quantity aircraft-months (ACM). Chapter III discussed the benefits of using ACM as a program element and described the calculation of the number of ACM for month t as $ACM_t = \sum_{i=1}^t (t + 1 - i) X_i$, where X_i = number of new aircraft in month i .

Consider the drawing of one aircraft-month from the aircraft-months calculated for month t (ACM_t) as a Bernoulli trial. The outcome of this trial is either a success, defined as a removal, or a failure, defined as a no removal. It is assumed that each ACM has a probability of experiencing one and only one removal which can be described by p_t , that is $p_t = \text{Pr} (1 \text{ removal in } 1 \text{ ACM})$.

A sum of Bernoulli trials can be described by a binomial model. Thus, it follows that the probability of observing k_t removals in n_t ACM's given a removal rate of p_t is,

$$\Pr(k_t | p_t) = \binom{n_t}{k_t} p_t^{k_t} (1-p_t)^{n_t-k_t} \quad (1)$$

and $E[k_t] = n_t p_t$ and $\text{Var}[k_t] = n_t p_t (1-p_t)$

The uncertainty which exists about the removal rate, p_t , is accounted for by assigning to it a probability distribution which summarizes designers', manufacturers' and support managers' best "guesses" as to what the removal rate will be when the WRA is in operation. The distribution of p is referred to as the prior distribution and is denoted by $F(p)$. The unconditional probability of observing k removals in n ACM is:

$$\Pr(k) = \int_0^1 \Pr(k|p) dF(p) = \int_0^1 \binom{n}{k} p^k (1-p)^{n-k} dF(p) \quad (2)$$

To this point, estimates of the distribution of removal data have been based solely on considerations prior to the generation of removal data. Modification of these prior beliefs as removal data are accumulated is accomplished by an application of Bayes Theorem as follows: Let

$f(p) = \frac{dF(p)}{dp}$ be the prior density and suppose that in each of n aircraft-months, X_i removals are observed, where X_i takes on the value zero or one and $i = 1, 2, \dots, n$. Then the conditional density of p , given the observations for a particular month is called the posterior distribution of p and is:

$$f(p|x_1, \dots, x_n) = \frac{\prod_{i=1}^n P_r(x_i|p) \cdot f(p)}{\int_0^1 \prod_{i=1}^n P_r(x_i|p) \cdot f(p) dp} \quad (3)$$

With the additional information about removal rate summarized by the posterior distribution, equation (2) now becomes,

$$P_r(k) = \int_0^1 \binom{n}{k} p^k (1-p)^{n-k} f(p|x_1, \dots, x_n) dp$$

2. Choosing a Prior Distribution

Raiffa and Schlaifer [11] establish the following criteria for choosing a suitable prior distribution $F(p)$:

- " a. F should be analytically tractable....
- b. F should be rich, so that there will exist a member of F capable of expressing the decision maker's prior information and beliefs.
- c. F should be parametrizable in a manner which can be readily interpreted, so that it will be easy to verify that the chosen member of the family is really in close agreement with the decision maker's prior judgments about $[p]$"

A random variable p is said to be distributed as the Beta distribution with parameters α and β [$\alpha > 0, \beta > 0$]

if

$$f(p) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha) \Gamma(\beta)} p^{\alpha-1} (1-p)^{\beta-1}$$

for $0 < p < 1$

then $E [p] = \frac{\alpha}{\alpha + \beta}$

and $\text{var } [p] = \frac{\alpha \beta}{(\alpha + \beta)^2 (\alpha + \beta + 1)}$

The Beta family of distributions is conjugate in Bayesian terms [12]. Thus, suppose X_1, X_2, \dots, X_n is a random sample from a Bernoulli distribution with an unknown value of the parameter p . Further, suppose that the prior

distribution of p is Beta (α, β) . Then, the posterior distribution of p , when $X_i = X_i$ ($i=1, \dots, n$) is

$$\text{Beta} \left(\alpha + \sum_{i=1}^n x_i, \beta + n - \sum_{i=1}^n x_i \right)$$

where n = number of ACM's

$$X_i = \begin{cases} 1 & \text{if a removal} \\ 0 & \text{if no removal} \end{cases}$$

Thus a Beta prior satisfies the tractability criterion since the posterior distribution is also in the Beta family and is easy to determine. The Beta family provides a wide range of shapes, amply satisfying the second criterion; and the Beta distribution is completely characterized by its mean and variance which are meaningful concepts to be used in estimating the value of p .

B. PARAMETER ESTIMATION

With the binomial model, the expected value of the number of removals for month t is $E \left[\sum k_t \right] = n_t p_t$. Thus the parameters to be estimated are n_t , the number of ACM's in month t , and p_t , the probability of one removal in one ACM in month t . The number of ACM's is estimated from the delivery schedule called for in the aircraft manufacturer's contract. This parameter can be estimated with great certainty because it is based on a legal document wherein changes in delivery schedules are usually known in advance. The estimation of p , however, is much less certain.

The mature removal rate (p =removals/ACM) does not occur instantaneously, but evolves over a period of time as shown in Figure 2. It is easily inferred that initial provisioning

decisions based upon mature removal rate estimates would result in an insufficient number of WRA's in inventory. The remainder of this section will develop a method to estimate the changing removal rate during the period of introduction of new aircraft into the Fleet.

1. Initial Estimate of p

Analysis of the data in Chapter III revealed that growth curves of the form $p = ae^{-bt}$ could be fit relatively well to the removal rates for the aircraft systems for which data were available. Under the assumption that the removal rates for all avionic WRA's generate this type of curve, it is reasonable to presume that the properties of the curves can be used to obtain reasonable estimates of p on new avionic WRA's introduced into the Fleet. Historical usage data of a similar WRA operating under the same environmental conditions anticipated for the equipment to be provisioned could be used to construct a removal rate curve. Regression analysis would then yield an equation for the removal rate in the form $p = ae^{-bt}$, which could be used to predict p. The problem is that there exists no quantitative rationale to consign a new WRA to a particular removal rate curve on an a priori basis.

a. Formulation of Prior Distribution

A point estimate of initial removal rates does not reflect the uncertainty about the estimate and/or the degree of similarity between the new and operational WRA. This uncertainty is reflected in the Beta distribution of the

prior estimate. Since the point estimate fixes the ratio $\frac{\alpha}{\alpha+\beta} = E [P]$, selection of α and β defines the entire prior distribution. If α is large, then β must also be large thus causing the standard deviation to be small. A large α and β yield a strong prior which is relatively difficult for observed data to modify. Small α yields a weak prior with a large standard deviation.

To illustrate the effect that the choice of α has on certainty, it is assumed that system 1 is a new system not yet introduced into the Fleet. It is further assumed that an estimate of the initial p was obtained from a similar operational equipment and is equal to 0.314. (This value is actually the mean of the initial values of p for the five A7E systems.) Thus, $\frac{\alpha}{\alpha+\beta} = 0.314$ and the choice of α will define the prior distribution. Table VIII displays various values of α and the resulting levels of certainty (standard deviations).

DEGREE OF CERTAINTY REFLECTED IN
PRIOR DISTRIBUTION

α	.01	.05	.1	.5	1.0	1.5	2.0	3.0	4.0	5.0
β	.023	.109	.218	1.09	2.18	3.27	4.36	6.54	8.72	10.9
Mean	.314	.314	.314	.314	.314	.314	.314	.314	.314	.314
S.D.	.453	.431	.403	.288	.227	.193	.171	.143	.125	.113

Table VIII

Figure 3 illustrates that the magnitude of the chosen α implies how certain the provisioner is about his initial point estimate of p and for the degree of similarity between the new WRA and operational WRA. In continuing the

above example, it is assumed that great uncertainty exists about the initial estimate of p and the degree of similarity between the new and operational equipments. Thus, $\alpha = 0.1$ is chosen which yields a very large standard deviation.

CHOICE OF α VARIES DEGREE OF CERTAINTY

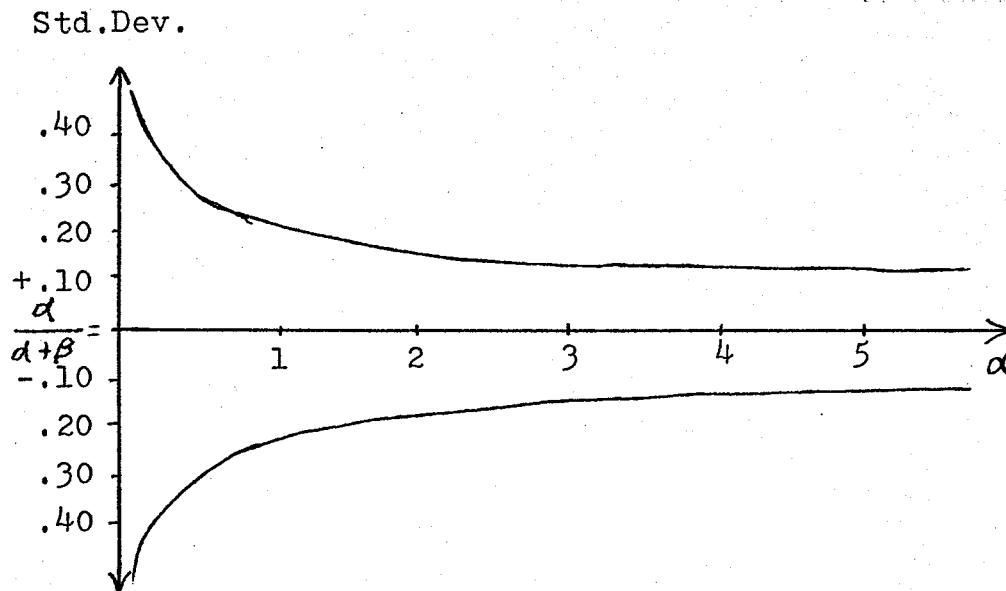


Figure 3

Informal rules for choosing α are as follows.

First determine the degree of similarity between the new WRA and an operational WRA. This may be accomplished by comparing the two in each of the following seven categories: (1) manufacturer's estimated mean time between failure (MTBF), (2) type function, (3) complexity, (4) state of technology, (5) fault isolation technique, (6) manufacturer's quality assurance, and (7) maintenance considerations. After these comparisons, then determine the degree of similarity between the two applications. If the two WRA's are thought to be

very similar, a value of α corresponding to a small standard deviation should be selected. Otherwise, an α indicating a great deal of uncertainty should be selected.

b. Adjusting the parameters b and a

The prior distribution for p has been described by choosing α and the parameters in the model $p = ae^{-bt}$ have been estimated by choosing a similar WRA and using its removal rate curve to predict future removals. The parameter b, however, can be adjusted to reflect the degree of management attention expected to be applied to correcting deficiencies in the newly provisioned WRA. An intense program dedicated to correcting deficiencies will be reflected in a rapidly decreasing removal rate.

Another consideration is the engineering of a WRA which is affected by the type of contract used for its procurement. Failure Free Warranty/Life Cycle Cost (FFW/LCC) method of procurement should enhance performance of new WRA's compared to conventional procurement and thus affect the rate of growth and hence the parameter b. The FFW/LCC procurement brings the manufacturer into an intimate loop which is dedicated to continuously upgrade field reliability of equipment.

The parameter "a" can be adjusted to reflect the belief that the point estimate of the initial removal rate is different from the initial rate of the similar operational WRA.

2. Updating the Estimate of p

Two sets of predictions seem to be needed. The first is made prior to aircraft introduction into the Fleet and is based on the experience of a similar operational WRA with the prior distribution on p . The second prediction is made after six months of experience is accumulated. This usage is summarized in the posterior distribution. The Bayes estimate of p (e.g. the expected value of the posterior-distribution) tends to smooth the data so that the estimate is probably closer to the then current removal rate. In addition, the Bayes estimate puts more weight on the latest observed value of p . Extrapolation from Bayesian estimates is more reasonable than from the erratic observed values of p until the observed values themselves have smoothed out sufficiently.

Extrapolation from month t through n months using the model $p = ae^{-bt}$ is accomplished by obtaining the Bayes estimate for p_t and using this as the estimate for a_t . The estimate of b is the original estimate obtained from the similar operational equipment adjusted, if necessary, to reflect a change in the slope of p . Predictions of p are then obtained by extrapolating the model $p_{t+i} = a_t e^{-bi}$, $i = 1, 2, \dots, n$.

To illustrate the updating of p , the example introduced in section 2 is continued. The initial estimate of p was 0.314 and $\alpha = 0.1$ was chosen to define the prior distribution on p . Figure 4 displays the observed values of p and the corresponding Bayesian estimates of p . The Bayesian estimate for month t is calculated as follows:

$$P_t^B = E[p_t] = \frac{\alpha_t}{\alpha_t + \beta_t}$$

$$\text{where } \alpha_t = \alpha_{t-1} + \sum_{i=1}^{n_t} \chi_i$$

$$\beta_t = \beta_{t-1} + n_t - \sum_{i=1}^{n_t} \chi_i$$

n_t = number of ACM's in month t

$\sum_{i=1}^{n_t} \chi_i$ = number of removals in month t

For early data in some of the systems the number of removals is greater than the number of ACM's. This violates the assumption of the binomial model and causes p to be greater than one. This was caused by the aggregation of WRA removals at the system level, and did not occur at the WRA level where actual provisioning is accomplished. Thus, the model is still valid for WRA provisioning. For purposes of the example, values of p in Figure 4 for months two and three were truncated at 0.99.

Figure 4 shows that while the initial estimate of $p = 0.314$ was extremely low compared to the observed value of 0.714, the Bayesian estimates reacted quickly to the data and smoothed the erratic nature of the observed values of p. The curves are converging because the variance is decreasing as ACM's increase.

It is desirable to compare early observed removal rates with prior predictions in order to determine whether the prior distribution is compatible with the observed removal rates [13]. An appropriate test of significance is given by the following procedure:

Let the predicted prior removal rate be, $p_t^B = \frac{\alpha}{\alpha + \beta}$.

Let the observed removal rate be,

$$p = \frac{\text{removals}}{\text{ACM}}$$

Construct a two sided 95.4% confidence interval

$$p_t - 2\sqrt{\frac{p_t(1-p_t)}{\text{ACM}}} < p_t^B < p_t + 2\sqrt{\frac{p_t(1-p_t)}{\text{ACM}}}$$

If p_t^B is included within the confidence interval, accept the hypothesis that p_t^B is compatible with the data and continue the Bayesian analysis. When p_t^B is outside the confidence interval, reject the hypothesis. The Bayesian prediction should then be discarded and the data estimate employed instead.

BAYESIAN UPDATE OF p

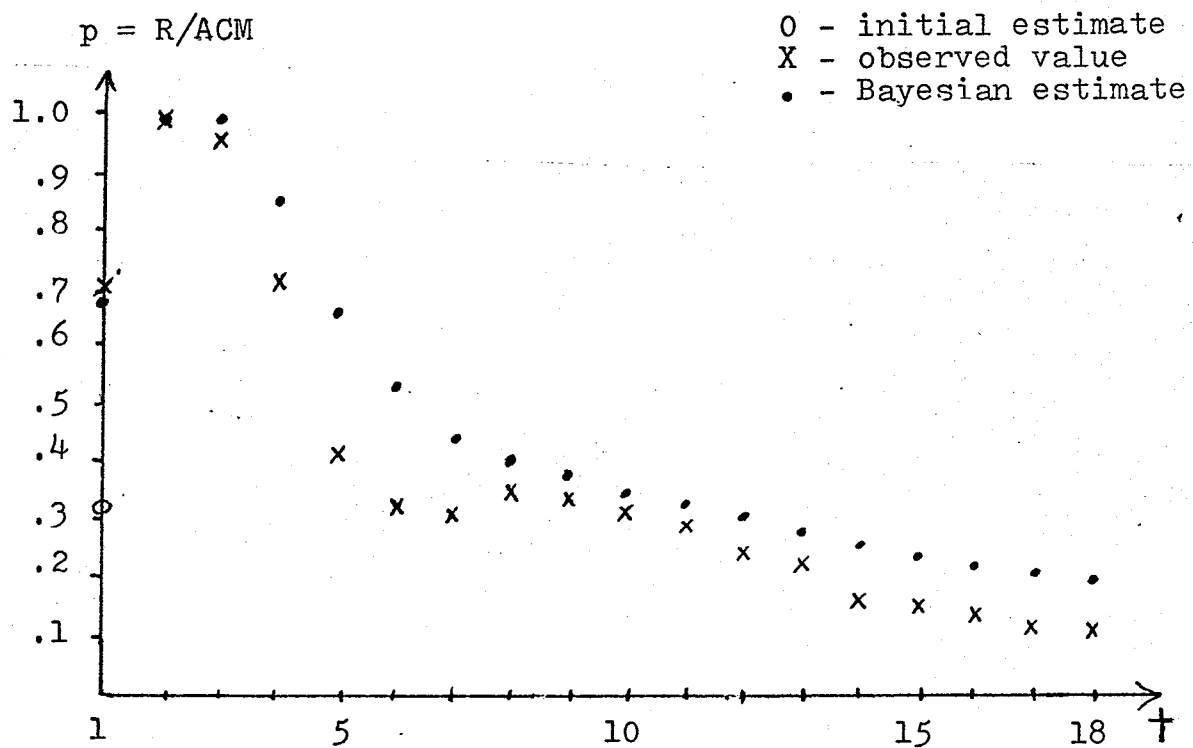


Figure 4

3. Sensitivity of Inventory Level to Changes in Parameter p

Figure 5 shows how various choices of the parameter p will affect the inventory level (I) of spare WRA's. In this example, a desired requisition effectiveness of 90 per cent is assumed. That is, if R_t = observed removals for month t , then it is required that $\Pr(R_t \leq I) = 0.9$. The normal approximation is used for the distribution of removals each month, so that R_t is distributed $N(\mu_t = n_t p_t, \sigma_t^2 = n_t p_t q_t)$. Thus the required inventory level for month t is $I_t = \mu_t + k \sigma_t$ where k is obtained from the standard normal distribution tables.

The parameter n_t , which is ACM's for month t , is the same in all three curves. The parameter p_t is varied, with the lower curve using values of p half as large as used in the middle curve, and the upper curve using values of p twice as large as the values used in the middle curve.

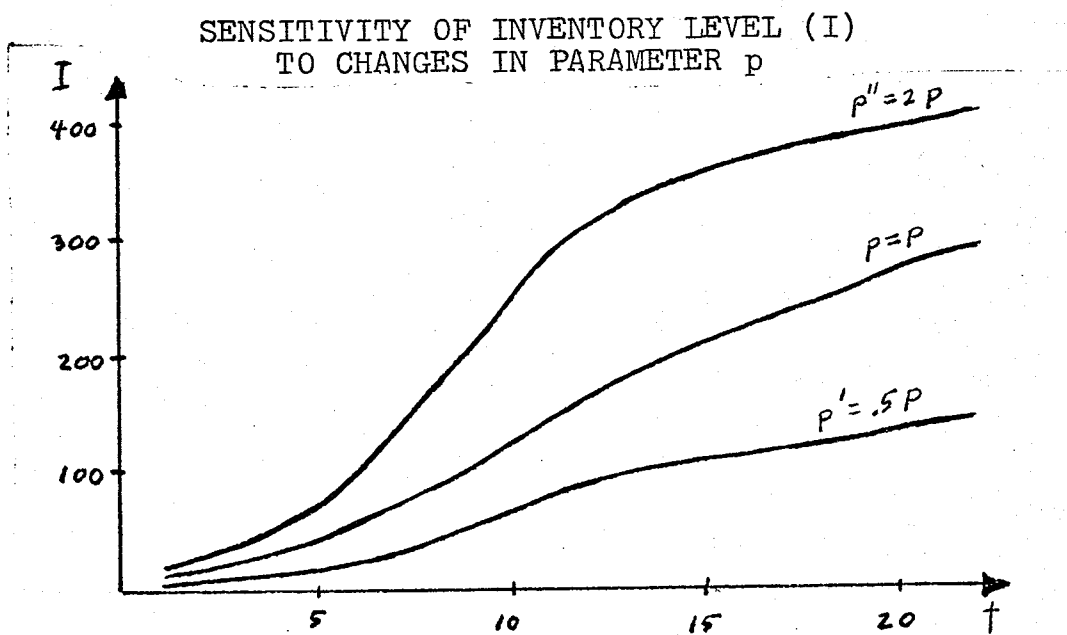


Figure 5

Figure 5 shows that inventory levels are quite close in the early months when there is great uncertainty about the parameter p . In the later months the inventory levels are diverging, when there is greater certainty about the parameter p .

V. SUMMARY AND CONCLUSIONS

A. SUMMARY OF THE MODEL

The binomial model developed to calculate the expected number of removals of Weapon Replaceable Assemblies (WRA) in a given month requires the monthly estimation of two parameters, aircraft-months (ACM) and the removal rate ($p = \text{removals}/\text{ACM}$). The parameter ACM is calculated from the manufacturers' aircraft delivery schedule delineated in the procurement contract, and the parameter p is estimated using a Bayesian technique and extrapolation.

The curve of p plotted over an initial provisioning period of eighteen months is modeled by the equation $p = ae^{-bt}$. The parameters a and b are estimated from a similar operational WRA and may be adjusted to reflect management responsiveness in supporting the WRA. The point estimate of the first month's p is assigned a prior distribution of Beta (α, β) which reflects the certainty of the point estimate and the degree of similarity between the new WRA and the similar operational WRA. The posterior distribution summarizes the accumulation of Fleet experience and combines this with predicted values for a unified estimate of the current month's true removal rate p_t . From this latest estimate of p_t , n monthly predictions are obtained for p by extrapolating the model, $P_{t+i} = a_t e^{-bi}$, $i=1, \dots, n$, where $a_t = p_t$. The expected number of removals in month t is the $E [k_t] = n_t p_t$, where $n_t = \text{ACM}_t$.

B. CONCLUSIONS

There exists no single formula that could usefully be employed across the board to estimate the number of WRA removals. This thesis has developed a method which can be tailored to a particular WRA for estimating future removal patterns. The application and limitations of this method are discussed in this section.

1. Prior Variance and Inventory Level

The immediate implication of a large prior variance, which reflects uncertainty about p , is that the probabilities of large demands and the probabilities of no demands increase. Thus, larger inventories are needed to provide desired system reliability, but the likelihood that these inventories will be wasted is also large. It may be concluded that a new weapons system whose performance may be projected only with great uncertainty will require a large inventory of spare WRA's.

2. Phased Procurement Alternative

The alternative to procuring large inventories with a potential for waste is to defer procurement until the accumulation of removal data permits more reliable predictions of removal rates.

For high-value, low-usage WRA's, the minimum-cost strategy might be to defer procurement until a significant number of removals occur. During the period of deferral, the Navy would buy the needed WRA's from a stock carried by

the manufacturer or from the production line if no stock is held. Reference 14 describes an algorithm for determining in what cases this would be the best policy.

3. Small Community Aircraft

Prediction models based on large numbers of aircraft can be inaccurate when applied mechanically to small community aircraft. The predictive method developed herein must be adjusted to account for the unique supply support required by small community aircraft, such as E-2B aircraft.

An alternative to high inventory levels and cannibalization required to support small community aircraft might be to allow contractors to provide the Naval Air Rework Facility functions of repair and to establish shore-based contractor teams to improve the organizational and intermediate maintenance capabilities. These types of actions would have to be reflected in the predictive model.

APPENDIX A

NON-AVIONIC SYSTEMS AND ASSEMBLIES

AC	System	Weapon Replaceable Assembly	Work Unit Code
F-14A	Wing Sweep Lateral Control Hydraulic	Inboard Spoiler Actuator	14231
		Mid wing Spoiler Actuator	14232
		Outboard Spoiler Actuator	14233
		Servoactuator Assy.	14234
		Tubing	14235
		Flexible Coil Tubing	14236
EA-6B	Hydraulic Pressure Source	Hydraulic Pump	45111
		Ram Air Turbine	45112
		Hydraulic Hand Pump	45113
		Hydraulic Accumulator	45114
		Parts not otherwise coded	45119

APPENDIX B

NON-AVIONIC SYSTEMS DATA

TABLE 1: AIRCRAFT: F-14A SYSTEM: LATERAL CONTROL HYDRAULIC

MONTH	REMOVALS	FAILURES	BCM	EFFECTIVE FLT.HRS	EFFECTIVE SORTIES	#AC
1	0	0	0	17	7	6
2	0	0	0	41	23	8
3	1	1	1	124	63	8
4	0	0	0	129	68	8
5	1	1	1	186	101	11
6	1	1	1	207	124	13
7	0	0	0	226	118	16
8	1	1	1	362	214	17
9	1	1	1	429	274	22
10	5	5	4	506	327	23
11	4	3	2	375	225	25
12	0	0	0	477	301	32
13	2	2	2	CLASSIFIED		
14	5	4	3			
15	1	1	1			
16	4	3	2			
17	1	0	0			
18	5	4	3			
19	1	1	0			
20	2	2	1			

TABLE 2: AIRCRAFT: EA-6B SYSTEM: HYDRAULIC PRESSURE SOURCE

MONTH	REMOVALS	FAILURES	BCM	EFFECTIVE FLT.HRS.	EFFECTIVE SORTIES	#AC
1	0	0	0	24 *	12 *	3
2	1	1	1	40 *	20 *	3
3	0	0	0	40 *	20 *	2
4	0	0	0	57	24	2
5	0	0	0	155	62	4
6	2	1	1	150 *	75 *	6
7	0	0	0	163 *	81 *	7
8	0	0	0	326	160	9
9	1	1	1	408	182	10
10	1	1	0	350	166	10
11	1	1	1	183	95	9
12	0	0	0	100 *	50*	10
13	0	0	0	294	148	11
14	2	2	2	524	273	11
15	1	1	1	540	325	12
16	4	4	0	341	170 *	13
17	1	1	1	412	206 *	13
18	5	5	4	487	278	14
19	1	1	0	313	178	16
20	1	1	1	320 *	160 *	18
21	3	3	3	334	185	19
22	4	3	2	505	273	20
23	2	2	1	498	262	21
24	10	8	6	421	200	21
25	3	1	1	612	277	21
26	3	3	1	671	337	23
27	1	1	0	936	435	23
28	1	1	0	616	302	23

* Indicates original data not available and quantity is estimated.

APPENDIX C

AVIONIC SYSTEMS AND ASSEMBLIES

System	A/C	System	Weapon Replaceable Assemblies	Work Unit Code
1	A7E	Forward Looking Radar AN/APQ -126(v)	AS2272 Ante/Revr T1091 Xmtr PP6130 Pwr. Supply CP954 Computer IP940 Indicator SG811 Sweep Gen. C7774 Set Control	73A11 73A12 73A13 73A14 73A15 73A16 73A17
2	A7E	Tactical Computer Set AN/ASN-91(v)	CP952 Tactical Comp. CP7830 Tact. Comp. Contr.	73A21 73A22
3	A7E	Inertial Measurement Set AN/ASN-90(v)	CN1260 IMS C7822 IMU Set Control PF6141 Pwr. Supply	73A51 73A53 73A54
4	A7E	Projected Map Display Set AN/ASN-99(v)	ID1665 Display Unit CV2622 Signal Data Converter	735A1 735A2
5	A7E	Head Up Display Set AN/AVQ-7(v)	IP946 Display CP951 Processor	73A41 73A42
6	P3C	Radio Set AN/ARC 143	RT832 Rcur/Xmtr C7791 Control	632k1 632k3
7	P3C	Radio Set AN/ARC 142	RT931 Rcur/Xmtr AM6114 RF Amplifier C7789 Control	612H1 612H2 612H3
8	P3C	Inertial Navigation Set AN/ASN 84	C7560 Gyro Control CN1231 Gyroscope C7561 Navig Control ID1542 Position Ind. PP4964 Pwr Supply CP924 Navig. Comp.	734F1 734F2 734F3 734F4 734F6 734F7
9	P3C	Search Radar Set AN/APS 115	AS2146 Antenna RT889 Revr/Xmtr MX7930 Ant Position Programmer C7512 Control C7511 Control	726A1 726A2 726A3 726A4 726A5

APPENDIX D
AVIONIC SYSTEMS DATA
TABLE 1 UTILIZATION DATA
Aircraft A-7E

Month	Effective Flt. Hrs.	Effective Sorties	#A/C	Aircraft Months
1	119	52	7	7
2	731	354	14	21
3	1,503	686	30	51
4	2,867	1,392	44	95
5	2,518	1,212	49	144
6	4,131	2,712	75	219
7	1,954	1,200	91	310
8	3,622	2,151	113	423
9	4,889	2,510	140	563
10	4,979	2,795	164	727
11	5,800	3,458	198	925
12	5,783	3,212	217	1142
13	6,553	3,818	245	1387
14	6,123	3,849	272	1659
15	8,559	4,887	262	1826
16	7,481	4,345	254	1988
17	7,305	4,513	265	2153
18	9,124	4,873	279	2532
19	7,732	4,124	267	2649
20	9,657	5,511	286	2935
21	12,459	6,781	303	3233
22	12,403	6,749	315	3553
23	11,530	6,098	311	3782
24	11,171	6,521	329	4111
25	8,048	4,717	306	4107
26	8,664	4,745	271	3872
27	8,176	4,536	268	4075
28	9,187	5,016	266	4316

Aircraft P-3C

Month	Effective Flt. Hrs.	Effective Sorties	#A/C	Aircraft- Months
1	64	22	3	3
2	114	44	6	9
3	396	100	10	19
4	562	142	11	30
5	730	182	12	42
6	716	192	14	56
7	926	264	16	72
8	1,616	420	20	92
9	2,124	556	23	115
10	3,090	714	34	149
11	2,828	818	29	138
12	2,688	698	29	167
13	2,912	718	36	203
14	2,900	708	39	242
15	3,678	772	40	282
16	4,132	938	42	324
17	4,100	870	44	368
18	5,388	1,136	49	417
19	4,622	978	49	466
20	4,100	904	51	517
21	5,752	1,150	53	570
22	6,606	1,302	53	623
23	5,772	1,198	62	685
24	5,030	982	58	659
25	7,144	1,292	60	719
26	8,824	1,640	63	782
27	9,618	1,756	65	847
28	6,534	1,190	68	915

APPENDIX D
TABLE 2 PERFORMANCE DATA

Aircraft A73

Month	Sys #1 APQ-126			Sys #2 ASN-91		
	Removals	Failures	BCM's	Removals	Failures	BCM'S
1	5	0	0	2	2	2
2	28	0	0	13	3	2
3	49	2	1	17	10	2
4	67	11	0	23	11	2
5	61	21	0	21	12	1
6	69	28	1	47	17	2
7	93	71	6	24	19	4
8	145	119	20	38	28	7
9	190	121	2	54	30	2
10	229	170	9	64	20	2
11	250	232	38	76	32	1
12	291	266	46	87	39	6
13	349	315	29	87	35	4
14	256	240	35	93	52	10
15	282	244	16	87	40	10
16	295	265	20	78	42	5
17	290	270	13	79	35	5
18	336	309	19	71	41	2
19	382	362	21	74	47	6
20	338	302	17	100	57	6
21	632	562	39	111	68	11
22	553	495	32	161	88	13
23	501	445	24	128	73	9
24	509	463	20	146	85	12
25	506	439	13	112	72	8
26	566	482	11	132	83	9
27	469	367	12	90	53	6
28	483	336	6	61	38	3

Aircraft A7E

Month	Sys #3 ASN-90			Sys #4 ASN-99		
	Removals	Failures	BCM's	Removals	Failures	BCM's
1	4	0	0	0	0	0
2	5	0	0	0	0	0
3	14	0	0	1	0	0
4	20	0	0	6	5	0
5	23	7	0	4	2	0
6	28	8	0	2	1	0
7	26	15	8	6	3	1
8	56	37	26	15	14	4
9	74	49	27	21	19	0
10	100	76	33	26	21	0
11	92	68	28	55	42	6
12	98	72	29	53	36	6
13	118	71	18	49	36	1
14	140	81	17	58	37	0
15	163	106	23	73	31	2
16	162	107	14	72	45	2
17	169	119	26	66	49	3
18	128	94	24	52	38	2
19	135	100	27	61	49	0
20	160	105	31	41	30	1
21	252	179	37	95	62	3
22	181	129	35	86	53	3
23	195	134	40	96	62	3
24	225	157	46	130	88	5
25	210	154	35	93	57	3
26	245	147	37	84	55	1
27	233	143	50	56	40	1
28	165	89	26	43	21	0

Aircraft A7E

Month	Sys #5	AVQ - 7	BCM's
	Removals	Failures	
1	0	0	0
2	11	0	0
3	12	0	0
4	12	4	0
5	18	8	0
6	30	15	0
7	34	17	0
8	47	28	3
9	50	17	0
10	82	45	1
11	102	57	2
12	85	59	9
13	119	90	4
14	125	88	13
15	124	75	16
16	104	27	4
17	145	65	6
18	166	78	4
19	193	87	4
20	169	97	22
21	172	120	58
22	158	135	56
23	119	110	42
24	147	115	31
25	136	105	38
26	148	112	26
27	126	94	18
28	113	70	11

Aircraft P3C

Month	Sys #6 ARC-143			Sys #7 ARC-142		
	Removals	Failures	BCM's	Removals	Failures	BCM's
1	16	4	0	13	6	1
2	10	3	1	19	13	1
3	2	1	0	24	16	3
4	0	0	0	16	14	1
5	2	2	0	18	16	3
6	16	8	2	21	14	4
7	9	8	0	37	28	3
8	14	9	0	31	23	0
9	23	13	1	35	22	4
10	29	22	0	65	39	3
11	28	18	0	68	33	1
12	21	3	1	49	3	0
13	29	23	7	78	50	23
14	22	13	3	63	40	24
15	26	15	2	56	37	14
16	22	18	2	54	39	22
17	50	41	6	60	45	19
18	40	30	8	42	26	11
19	45	27	6	62	45	18
20	38	23	7	51	34	13
21	42	37	5	57	46	23
22	48	39	14	59	38	15
23	43	33	10	70	47	10
24	43	34	11	58	35	16
25	39	27	6	57	39	10
26	40	27	14	87	59	21
27	42	31	10	60	41	16
28	34	23	6	55	31	3

Aircraft P3C

Month	Sys #8	ASN-84		Sys #9	APS-115	
	Removals	Failures	BCM's	Removals	Failures	BCM's
1	55	9	2	9	4	3
2	60	19	2	9	3	1
3	22	19	2	10	5	2
4	43	33	1	17	11	4
5	43	35	4	18	12	7
6	55	32	1	11	7	6
7	51	37	7	21	15	3
8	75	47	4	25	21	6
9	129	61	16	21	19	9
10	149	73	10	32	26	12
11	101	51	14	29	21	11
12	73	24	5	26	13	2
13	98	75	12	33	19	4
14	69	52	12	38	22	6
15	82	51	14	38	21	3
16	103	71	13	41	24	7
17	86	61	11	28	19	5
18	88	60	11	33	26	6
19	78	54	16	45	27	1
20	77	60	15	30	24	5
21	83	61	11	39	28	4
22	92	60	9	36	23	11
23	117	87	19	31	18	3
24	82	64	21	48	29	11
25	65	49	14	42	32	10
26	96	74	36	62	31	8
27	74	45	14	74	49	16
28	98	62	28	50	30	12

APPENDIX E

TEST OF PARAMETERS b_i SIMILARITY

Table 1

$H_0: b_i = b$ for 5 systems in A7E

H_1 : otherwise

where b_i = coeff. of x as in $y = a + b_i x$
 $i = 1, 2, 3, 4, 5$

$$b = \frac{1}{5} \sum_{i=1}^5 b_i = -.062$$

Tabulated value of t statistic = 2.056

System i	$t = \frac{b_i - b}{\sqrt{\text{Var}[b_i]}} = \text{Test Statistic}$	Accept H_0 - A Reject H_0 - R
1	$\frac{(-0.075) - (-.062)}{.007} = -1.857$	A
2	$\frac{(-0.103) - (-.062)}{.008} = -5.125$	R
3	$\frac{(-0.067) - (-.062)}{.007} = -.714$	A
4	$\frac{(-0.022) - (-.062)}{.012} = 3.333$	R
5	$\frac{(-0.043) - (-.062)}{.012} = 1.583$	A

APPENDIX E

Table 2

$H_0: b_i = b$ for 4 systems in P3C

H_1 : otherwise

where b_i = coeff. of x as in $y = a + b_i x$
 $i = 6, 7, 8, 9$

$$b = \frac{1}{4} \sum_{i=1}^4 b_i = -.115$$

System i	$t = \frac{b_i - b}{\sqrt{\text{Var } b_i}} = \text{Test Statistic}$	
6	$\frac{(-.090) - (-.115)}{.019} = 1.316$	A
7	$\frac{(-.118) - (-.115)}{.010} = -.3$	A
8	$\frac{(-.143) - (-.115)}{.012} = -2.333$	R
9	$\frac{(-.107) - (-.115)}{.010} = .8$	A

APPENDIX F

TEST OF REGRESSION SIGNIFICANCE

$$H_0: b_i = 0$$

H_1 : otherwise

where b_i is in $\ln y_i = \ln a_i + b_i t$

Tabulated value of statistic = 2.056

System i	$t = \frac{b_i - 0}{\sqrt{\text{Var } b_i}} = \text{Test Statistic}$	Accept H_0 - A Reject H_0 - R
1	$\frac{-0.075}{.007} = -10.7$	R
2	$\frac{-0.103}{.008} = -12.9$	R
3	$\frac{-0.065}{.007} = -9.3$	R
4	$\frac{-0.022}{.012} = -1.8$	A
5	$\frac{-0.043}{.012} = -3.6$	R
6	$\frac{-0.90}{.019} = -4.7$	R
7	$\frac{-0.118}{.010} = -11.8$	R
8	$\frac{-0.143}{.012} = -11.9$	R
9	$\frac{-0.107}{.010} = -10.7$	R

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